RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SIXTH SEMESTER EXAMINATION, MAY 2016

THIRD YEAR [BATCH 2013-16]

Date : 14/05/2016 MATHEMATICS (Honours)

Time : 11 am – 3 pm **Paper :** VIII Full Marks : 70

[Use a separate Answer Book for each Unit]

Unit - I

Answer any five questions : [5×6]

1. The couple components of a system of coplanar forces when reduced with respect to two different bases O and O' are G and G' respectively. Show that the couple component when the system is reduced with respect to the middle point of OO' is $\frac{1}{2}(G+G')$.

2. A frame consists of five bars forming the sides of a rhombus ABCD and the diagonal AC. If four equal forces P act inwards at the middle points of the sides and at right angles to the respective sides, prove that the tension in AC is $\frac{P\cos 2\theta}{\sin\theta}$, where $\theta = \angle BAC$. The bars are weightless.

3. Four uniform rods AB, BC, CD, DE each of length 'ℓ' and weight 'w' are smoothly joined at their ends B, C and D and ends A and E are smoothly joined to two fixed points at distance '2ℓ' apart in the same horizontal line. If AB, BC make angles 'α' and 'β' respectively with the horizontal when the system hangs in equilibrium, show, by the principle of virtual work, that $3\cot\alpha = \cot\beta$, B and D being connected by an inextensible string of length ℓ.

4. A perfectly rough heavy body rests in equilibrium on a fixed body. Considering the surface of the fixed lower body is convex upwards and the upper body is in equilibrium at the highest point of the fixed body, discuss the stability of the equilibrium of the upper body.

5. A, B are two points on the same horizontal level at a distance a. A uniform rod AC of length a and weight $2\sqrt{3}$ lb. is hinged at A and is supported by a string fastened at C, passing over a smooth pulley at B and carrying a weight of one pound. Find the position of equilibrium and show that it is stable.

6. Three forces, each equal to P act on a body— one at the point (a, 0, 0) parallel to OY, second at a point (0, b, 0) parallel to OZ and the third at point (0, 0, c) parallel to OX, the axes being rectangular. Show that the central axis passes through the point
$\left(-\frac{a+2b+3c}{3}, -\frac{b+2c+3a}{3}, -\frac{c+2a+3b}{3}\right)$ and has its direction ratios 1,1,1.

7. Two equal uniform ladders are jointed at one end and stand with other ends on a rough horizontal plane. A man whose weight is equal to that of the ladders ascends one of them. Prove that the other will slip first. If it begins to slip when he has ascended a distance x, prove that the coefficient of friction is $\left(\frac{a+x}{2a+x}\right)\tan\alpha$, where a being the length of each ladder and α being the angle which each makes with the vertical.

8. A uniform heavy string rests on a smooth parabola, whose axis is vertical and vertex upward, so that its ends are at the extremities of the latus rectum. Show that the pressure on the curve, at the point where the tangent makes an angle ϕ with the horizontal, is $\frac{w}{2}(2\cos^3\phi + \cos\phi)$, where w is the weight per unit length of the string.

Unit - II

***Answer any two questions* :** [2×10]

9. a) Design a circuit so that a light will glow when a majority of votes is cast in favour of a proposal from four voters who have voting weights of 4, 3, 1, 1 respectively. [4]

   b) Express the Boolean expression $(x+y+z)(xy+x'z)'$ in the variables x, y, z in DNF (Disjunctive Normal Form). [3]

   c) Describe the output generated by the following C program. [3]

```
int funct (int x)
    {
        static int y = 0;
            y + = x;
            return (y);
    }
main ( )
    {
        int a, count;
        for (count = 1; count < = 5; ++count)
            {
                a = funct (count);
                printf (“% d”, a);
            }
    }
```

10. a) In a Boolean algebra $(B,+,\cdot,')$ prove that $a+(b+c)=(a+b)+c, a\cdot(b\cdot c)=(a\cdot b)\cdot c$, for all $a,b,c\in B$. Deduce that $a+b=0 \Rightarrow a=0, b=0$. [3]

    b) A C program contains the following declaration

    static int x[8] = {10, 20, 30, 40, 50, 60, 70, 80};

    i) What is the meaning of x?
    ii) What is the meaning of (x+2)?
    iii) What is the value of $*x$?
    iv) What is the value of (*x+2)?
    v) What is the value of *(x+2)?
    vi) What is the value of *(& x[6])? [3]

    c) Write a C program to find the sum of the series $\frac{x}{1!}-\frac{x^3}{3!}+\frac{x^5}{5!}-$ ...... for a given value of x = x_0 such that $-1\le x_0\le 1$, correct upto 5 decimal places. [4]

11. a) What is the purpose of the switch statement? What are case labels? What are differences between passing an array to a function and passing a single valued data item to a function? [3]

    b) What is the purpose of the break statement? Suppose a break statement is included within the innermost of several nested control statements. What happens when the break statement is executed? Compare the continue statement with the break statement. [3]

c) Describe the output generated by the following program. [4]

```
# include <stdio.h>
int funct2 (int a)
    {
        static int b=1;
            b +=1;
            return (b+a);
    }
int funct1 (int a)
    {
        int b;
        b = funct2(a);
        return (b);
    }
main ( )
    {
        int a=0, b=1, count;
        for (count = 1; count <=5; ++ count)
            {
                b +=funct1(a) + funct2(a);
                printf (“% d”, b);
            }
    }
```

[Attempt either Unit – III or Unit – IV]

Unit – III

Answer any four questions : [4×5]

12. a) If A^i_j are components of a mixed tensor of type (1,1) and u_i and v^j are respectively a covariant and contravariant vectors then show that $A^i_j u_i v^j$ is an invariant. [3]

b) Show that in the Riemannian space V_4 with line element

$(ds)^2 = -(dx^1)^2 - (dx^2)^2 - (dx^3)^2 + c^2(dx^4)^2$, the vector $\left(1, 1, 0, \frac{\sqrt{3}}{c}\right)$ is a unit vector. [2]

13. Prove that $-g^{kj}\left\{ \begin{matrix} \ell \\ ij \end{matrix} \right\} - g^{\ell j}\left\{ \begin{matrix} k \\ ij \end{matrix} \right\} = \frac{\partial g^{k\ell}}{\partial x^i}$, where the symbols have their usual meaning. [5]

14. a) If $B^i_{jk} = \delta^i_j \frac{\partial \phi}{\partial x^k} - \delta^i_k \frac{\partial \phi}{\partial x^j}$, where ϕ is an invariant, prove that $g^{jk} B^i_{jk} = 0$. [2]

b) Prove that $g_{ij,k} = 0$. [3]

15. a) If A_i is a covariant vector show that $\frac{\partial A_i}{\partial x^j}$ is not a tensor. [1]

b) If A^{ij} are components of a symmetric contravariant tensor of rank 2, then show that $A^{jk}[ij, k] = \frac{1}{2} A^{jk} \frac{\partial g_{jk}}{\partial x^i}$. [4]

16. a) If a tensor $A_{ijk\ell}$ is symmetric in the first two indices from the left and skew-symmetric in the second and fourth indices from the left, show that $A_{ijk\ell} = 0$. [2]

b) If A^i and B^i are two non-null vectors such that $g_{ij}U^iU^j = g_{ij}V^iV^j$ where $U^i = A^i + B^i$ and $V^i = A^i - B^i$, show that A^i and B^i are orthogonal. [3]

17. Prove that the transformations of covariant vectors form a group. [5]

Unit - IV

Answer any four questions : [4×5]

18. Prove that the Sorgenfrey line is first countable but not second countable. [2+3]

19. a) Give an example of a sequence in a topological space, which converges to infinitely many points. [2]

b) Is Lindelöfness a hereditary property? Support your answer. [3]

20. Give an example of a function $f:(X,\tau) \to (Y,\tau_1)$ from a topological space (X,τ) to a topological space (Y,τ_1) such that for every sequence $\{x_n\}_{n\in\mathbb{N}}$ in (X,τ) converging to a point $a \in X$, the sequence $\{f(x_n)\}_{x\in\mathbb{N}}$ converges to f(a), but f is not continuous. [5]

21. Let $p:\mathbb{R} \to \mathbb{R}$ be a polynomial. Show that p is a closed map. [5]

22. Prove that $S^1 = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ and $T^1 = \left\{(x,y) \in \mathbb{R}^2 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\right\}$ are homeomorphic. [5]

23. Prove that every topological space is a subspace of a seperable space. [5]

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